A Mixed Historical Formula to forecast volatility

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Abstract

This study presents a new methodology for forecasting volatility. It relies on a weighted mean of short and long estimates of variance, based on a Moving Average framework. The quality of the predictions obtained with the proposed formula was checked with both simulated and real data. When applied to the analysis of simulated data, the new formula provides the least reliable forecast when a Random Walk is used as Data Generating Process (DGP) and the forecast variance is a simple Moving Average or when the DGP belongs to the ARCH model family and the associated forecast formula is used. However, compared to existing approaches, the new methodology allows for the most reliable forecast on 5-day and 20-day horizons, when it is applied to Index, Fixed Income and Foreign Exchange data series.

Key words: Volatility forecasting, GARCH models, Evaluating forecasts,
Non-parametric methods, Exponential smoothing

1 Introduction

Since the introduction of the CAPM and Black-Scholes models, volatility has played a crucial role in the definition of derivative price. However, volatility is both a *random* and an *unobserved variable*, that must be inferred from data. Several models have been proposed to estimate volatility. They typically use either historical data (historical volatility) or option market prices (implied volatility). A complete review of volatility models used for forecasting can be found in Poon and Granger (2003). Many subsequent papers have attempted to improve the forecasting of volatility and have also introduced the practice of checking the quality of the forecast. The work of Hansen and Lunde (2005) and McMillan and Speight (2004) confirms the superiority of the ARCH model over all other models as far as daily forecasts are concerned. The quality of their predictions is due to using cumulative squared returns from intra-day data (realized volatility) rather than daily squared returns in comparison with the variance forecast. This new method of measuring volatility was recommended by Andersen and Bollerslev (1998). In order to reduce the high persistence typical of the GARCH model, Marcucci (2005) introduces a new regime-switching GARCH model, that gives optimal results for 1-day forecasts. Koopman et al. (2005) compare historical and implied volatility on the S&P100 stock index. Their results show that historical volatility allows for better results on a 1-day horizon and that the best performing model is an ARFIMA model. Gonzalez-Rivera et al. (2004) instead, introduce four different loss functions (two from the statistical world and two from the economic world) to evaluate the quality of forecasting. Their results seem contradictory at first. However, the four measures used are very different, and it is very

difficult to understand the role played by volatility in each of the measures.
This work deals with the Moving Average model. The best-known model is Taylor's Exponential Weighting Moving Average (EWMA) Taylor (1986). Taylor proved that the EWMA model could produce superior forecast predictions using a short (10 days) time horizon, because it requires fewer parameters to be estimated and it is the most sensitive to changes. Akgiray (1989), using different series and a longer time horizon (20 days), showed that the GARCH(1,1) model could produce better predictions than the EWMA model. Many authors agree that the EWMA model is the best available model for forecast (Tse, 1991; Tse and Tung, 1992; Boudoukh et al., 1997; Walsh and Tsou, 1998). The EWMA model became widely known when RiskMetrics (1996) introduced it in their important work on market risk measure. A variant of the EWMA model was proposed by Taylor (2004), where, using a logistic function, the weight depends on the past returns.
In this work the results of a new forecasting method based on a Moving Average model formulation are presented. Our method is based on a weighted mean of a short and a long estimate of historical variance. The short estimate is based on an EWMA model, while the long estimate is based on an MA model. The method performs well regardless of the data generation processes used, and it obtains better results on a short horizon (within 1 month) than classical models (EWMA, GARCH(1,1)) when tested on market index, foreign exchange and fixed income series. Additionally, our methodology is *very easy to implement*.
The paper is organized as follows. Section 2 is devoted to introducing four classical volatility forecasting models that will be used as benchmarks. The new proposed methodology is presented in section 3. The procedure used to test volatility forecasting is explained in section 4. The main results from simulated and real data are shown in sections 5 and 6, respectively. In section 7, conclusions are drawn.

1.1 Notation

Let p_t $(t = 1, \dots, T)$ be an asset price defined on $(\Omega, \mathscr{F}_t, \mathbb{P})$, where $\mathscr{F}_t$ is the natural filtration associated with p_t. Let $r_t = ln(p_t) - ln(p_{t-1})$ be the continuously compounded return on the asset over the period $t-1$ to t. We break down $r_t = \mu_t + \sqrt{h_t}\epsilon_t$, where $\mu_t = \mathbb{E}[r_t|\mathscr{F}_{t-1}] = \mathbb{E}_{t-1}[r_t]$ is the conditional mean, $h_t = \mathbb{E}[(r_t - \mu_t)^2|\mathscr{F}_{t-1}] = \mathbb{E}_{t-1}[(r_t - \mu_t)^2]$ is the conditional variance and ϵ_t is a stochastic process defined on the same $\mathscr{F}_t$. We indicate the unconditional mean with $\mu = \mathbb{E}[r_t]$ and the unconditional variance with $\sigma^2 = \mathbb{E}[(r_t - \mu)^2]$. In this paper we assume $\mu = \mu_t = 0$.

2 Volatility models

2.1 GARCH(1,1)

The GARCH(1,1) model was introduced by Bollerslev (1986). The conditional variance is defined as $h_{t+1} = \alpha_0 + \alpha_1 r_t^2 + \beta_1 h_t$, where α_0, α_1 and β_1 are positive. To satisfy the hypotheses of stationarity, it is sufficient that $\alpha_1 + \beta_1 < 1$. The innovation terms, ϵ_t, are independent and identically distributed with null mean and unitary variance.

The variance forecasting at τ steps is given recursively and can be expressed in closed form as

$$h_{t+\tau} = \sigma^2 + (\alpha_1 + \beta_1)^{\tau-1} \left(h_t - \sigma^2\right) \tag{1}$$

where

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta_1}. \tag{2}$$

It can be observed that for high values of τ, i.e. for long horizon forecasts, the conditional variance is equal to the unconditional variance of the model.
We define the aggregated variance forecasting at τ steps as $h_{t+1:t+\tau} = \sum_{i=1}^{\tau} h_{t+i} = \tau\sigma^2 + \frac{(1-\alpha_1-\beta_1)^\tau}{1-(\alpha_1+\beta_1)} \left(h_t - \sigma^2\right)$

2.2 GJR-GARCH(1,1,1)

The GJR-GARCH(1,1,1) was introduced by Glosten et al. (1993) because the conditional variance shows some non-symmetric features such as a leverage effect. The conditional specification is $h_{t+1} = \alpha_0 + \left(\alpha_1 + \gamma_1 1_{r_t>0}\right) r_t^2 + \beta_1 h_t$, where $\alpha_0, \alpha_1, \beta_1$ and γ_1 are positive. To satisfy the hypotheses of stationarity, it is sufficient that $\alpha_1 + \gamma_1/2 + \beta_1 < 1$. The innovation terms, ϵ_t, follow the same condition as in the GARCH(1,1) model.

The variance forecasting at τ steps is given recursively and can be expressed in closed form as $h_{t+\tau} = \sigma^2 + \left(\alpha_1 + \frac{\gamma_1}{2} + \beta_1\right)^{\tau-1} \left(h_t - \sigma^2\right)$ where $\sigma^2 = \frac{\alpha_0}{1-\alpha_1-\frac{\gamma_1}{2}-\beta_1}$.

2.3 Moving Average

The Moving Average (MA) model is a non-parametric model where the variance at time t is obtained from equally weighted historical observations, i.e. $h_t =$

$\frac{1}{p}\sum_{j=0}^{p-1} r_{t-j}^2$, where p is the window length. In non-parametric models, the conditional variance at τ steps is simply given by $h_{t+\tau} = h_t$ and the aggregated variance forecasting at τ steps as $h_{t+1:t+\tau} = \tau h_t$

2.4 Exponential Weighted Moving Average

The Exponential Weighted Moving Average (EWMA) is a moving average of historical observations where the latest observations carry the highest weight in the variance estimate,

$$h_t = \frac{\sum_{j=0}^{p-1} \Phi^j r_{t-j}^2}{\sum_{j=0}^{p-1} \Phi^j} \quad \Phi \in [0, 1].$$

This approach has two important advantages over the equally weighted model. Firstly, the conditional variance is more responsive to shocks in the market as recent data has more weight than data in the distant past. Secondly, following a shock (a large return), the conditional variance declines exponentially as the weight of the shock observation falls.

The EWMA model has been identified by RiskMetrics (RiskMetrics, 1996) as the best forecasting model on both 1-day and 1-month horizons for interest rates, market indexes and foreign Exchanges rates.

It is has been proved that forecasting conditional variance of a I-GARCH(1,1) model with $\beta \approx 1$ is quite equivalent to an EWMA with $\Phi = \beta_1$.

3 A Mixed Historical Formula (MHF) for variance forecasting

The proposed formulation originates from two main observations. The first observation pertains to the variance forecasting of a GARCH(1,1) model (eq. 1). The variance of a GARCH(1,1) model shows an interesting feature, as it is a weighted sum of conditional variance h_t and an unconditional variance σ^2. Such a definition allows us to reinterpret the variance of a GARCH(1,1) model as a weighted mean of a short and a long estimated variance. However, the variance depends strongly on $\alpha_1 + \beta_1$, which one can assume to be close to 1 (I-GARCH effect), thus resulting in a biased estimation of the unconditional variance and consequently of the variance forecasting.

The second observation stems from the work of Mikosch and Stărică (2004). They proved that the I-GARCH effect is caused by abrupt changes in the unconditional variance. Subsequently, Stărică and Granger (2005) have reported observing this feature in many financial series. The main issue lies in the fact that the GARCH(1,1) model requires many points (at least 1000 or 4 years of daily observation) to reach stable convergence of the algorithm with narrow

confidence bands. In such a long period, there could be disruption to the unconditional variance.
The *Mixed Historical Formula* model we are proposing uses the forecasting formula provided by the GARCH(1,1) model, but adopts a Moving Average approach to estimate short and long variances. As such, it does not need many points and it is expected not to be affected by abrupt changes in the unconditional variance. The weights for the short and long variances used in the GARCH(1,1) model are here functions of a parameter ρ, chosen in a set $[0.87, 0.99]$. The forecasting of variance at τ steps then becomes

$$\sigma^2_{t+\tau} = \sigma^{2,long}_t \left(\rho^{\tau-1} - 1\right) + \rho^{\tau-1}\sigma^{2,short}_t \tag{3}$$

where $\sigma^{2,long}_t$ is obtained by a MA model carried out on the last 500 points, while $\sigma^{2,short}_t$ is obtained by an EWMA model on the last 70 points with $\Phi = 0.97$. The mean variance of a period $[t+1, t+\tau]$ is of great interest to market applications and it can be expressed as

$$\sigma^2_{t+1:t+\tau} = \frac{1}{\tau}\sum_{i=1}^{\tau}\left[\sigma^{2,long}_t \left(\rho^{i-1} - 1\right) + \rho^{i-1}\sigma^{2,short}_t\right] \tag{4}$$

One could argue that setting constant values for the parameters is too strong of assumption. We believe that such a practice is well founded. If series are not stationary, in-sample estimate of parameters cannot be efficient for forecasting, and, in order to estimate parameters, techniques such as regression, or similar must be used. Such techniques require many points (more than 1000 based on experience with ARCH models) and we would incur the same difficulty we are trying to avoid. The values chosen for the parameters are in fact consistent with our assumptions. If a sample of 500 points (two years of daily observations) is used to estimate $\sigma^{2,long}_t$, it should be long enough to guarantee a correct estimate of mean variance and it should not be affected by the potential presence of abrupt changes.

The choice of $\Phi = 0.97$ in the EWMA model is based on the work of RiskMetrics (1996). They empirically prove that this value is an optimum value for a one month horizon. The main difference between our work and the work of RiskMetrics (1996) is that they effectively employ 550 points - because for further values the weight is close to zero - while we have opted to use only the last three months of daily observation, i.e. a genuine short variance.
The proposed formula has two benefits: *(i)* as opposed to the EWMA and MA models, it produces good predictions on both short and long horizons and *(ii)* it does not require any estimation, because it does not belong to the class of parametric models.

4 Set-up to compare forecasts

This section describes the methodology used to compare the forecasting performance of the different models described in the previous sections.
Given a sample of length N, we use a subsample composed of first n points to obtain a variance forecasting on the next τ points. Then we compare this value with the historical variance obtained on the sub-sample $[r_{n+1} : r_{n+\tau}]$. We move Δ points forward and, using a sub-sample of n points, we repeat the procedure until the end of the sample. Hence, we obtain a $K = \frac{N-n-\tau}{\Delta}$ variance forecasts, which we compare to the historical one. To check the quality of variance forecasting, we use a Minimum Squared Error (MSE), thus defined

$$MSE(\tau) := \frac{1}{K} \sum_{k=0}^{K-1} \left(\bar{r}^2_{n+k\Delta+1:n+k\Delta+\tau} - h_{n+k\Delta+1:n+k\Delta+\tau} \right)^2 \tag{5}$$

$$\bar{r}^2_{t+1:t+\tau} := \sum_{1=1}^{\tau} r^2_{t+i} \tag{6}$$

This definition of MSE is preferred to a simpler one

$$MSE(\tau) := \frac{1}{K} \sum_{k=1}^{K-1} \left(r^2_{n+k\Delta+\tau} - h_{n+k\Delta+\tau} \right)^2$$

since the latter uses a poor measurement of the historical return variance.
The results are then presented as the ratio $MSE^{model\,1}(\tau)/MSE^{model\,2}(\tau)$. A ratio greater than unity at horizon τ indicates that the variance forecast of model 2 is better than variance forecast of model 1.
Given daily data, we have chosen to compare the MHF on two significant horizons: one week ($\tau = 5$) and one month ($\tau = 20$). As explained by RiskMetrics (1996, p.87), the multiple day forecast of MA-type models is based on a time scale, but "the *square root of time rule* results from the assumption that variances are constant". So, as they conclude: "scaling up volatility estimates prove problematic ... when estimates of volatilities optimized to forecast changes over a particular horizon are used for another horizon (jumping from daily to annual forecasts, for example)".

5 Simulated analysis

We now assess the merit of the proposed modeling approach using simulated data. If we assume the data generating process (DGP) to be known, we expect the best forecasting is to be produced by the formula associated to that DGP. Additionally, for the Mixed Historical Formula to produce acceptable results with real data, first it needs to produce satisfactory results with simulated data. Furthermore, simulations can lead to a better understanding of the role of ρ.

The DGPs involved in the simulation are GARCH(1,1), GJR-GARCH(1,1,1) and Random Walk.
Due to the number of different models, choice of parameters and horizon forecast, a large amount of data is produced. We wish to make the data available to those who have a particular interest in specific subsets. In this article, results are presented both extracting representative subsets as well as describing results for the data not shown here.

5.1 ARCH family DGP

We simulate 250 paths, each containing $N = 7020$ points. We then use the models for a sub-sample of length n belonging to a subset $\{500, 1000, 2000\}$, since we expect the forecast to improve with a longer sub-sample. The rolling sub-sample is moved forward of $\Delta = 10$ points. The last $\tau = 20$ points are left only for out-of-sample forecasting comparison. The number of forecasts K for each path is 500.
The simulations are performed with different choices of parameter, as shown in figure 1. We analyze three different regions: a region very close to the border line of stationarity (I-GARCH region) (region 1); a region with $\alpha_1 + \beta_1 \approx 0.97$ where the I-GARCH effect is not present, but the persistence of variance is high (region 2) and a region with $\alpha_1 + \beta_1 \approx 0.9$ with low persistence (region 3) [1].
All the results are presented for the GARCH(1,1) model, because the results for the GJR-GARCH(1,1,1) model are very similar.

For each choice of parameter, we calculate the 5^{th} percentile and the 95^{th} percentile of the empirical distribution of the ratio MSE^{GARCH}/MSE^{MHF}. Figure 2 shows the results for the three regions and for two different forecast horizons with $n = 2000$ and $\rho = 0.95$. It is apparent that the range between the lower and the higher percentile is almost always lower than unity, i.e. the GARCH(1,1) model obtains lower MSE, as expected. The MHF model produces poor results when the true DGP is a GARCH(1,1) model without I-GARCH effect (second and third line in figure 2). Finally, one observes that it is very rare for the ratio to exceed 1.5. However, as will be shown next, this

[1] For GJR-GARCH(1,1,1) model, the three regions, reported in figure 1 right panel are similar, but instead of $\alpha_1 + \beta_1$, we have $\alpha_1 + \beta_1 + \gamma_1/2$.

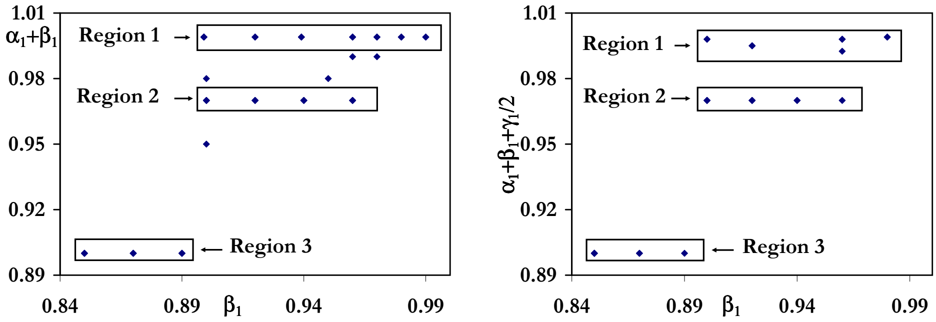

Fig. 1. Parameters used in simulations for a GARCH(1,1) model (left panel) and GJR-GARCH(1,1,1) model (right panel).

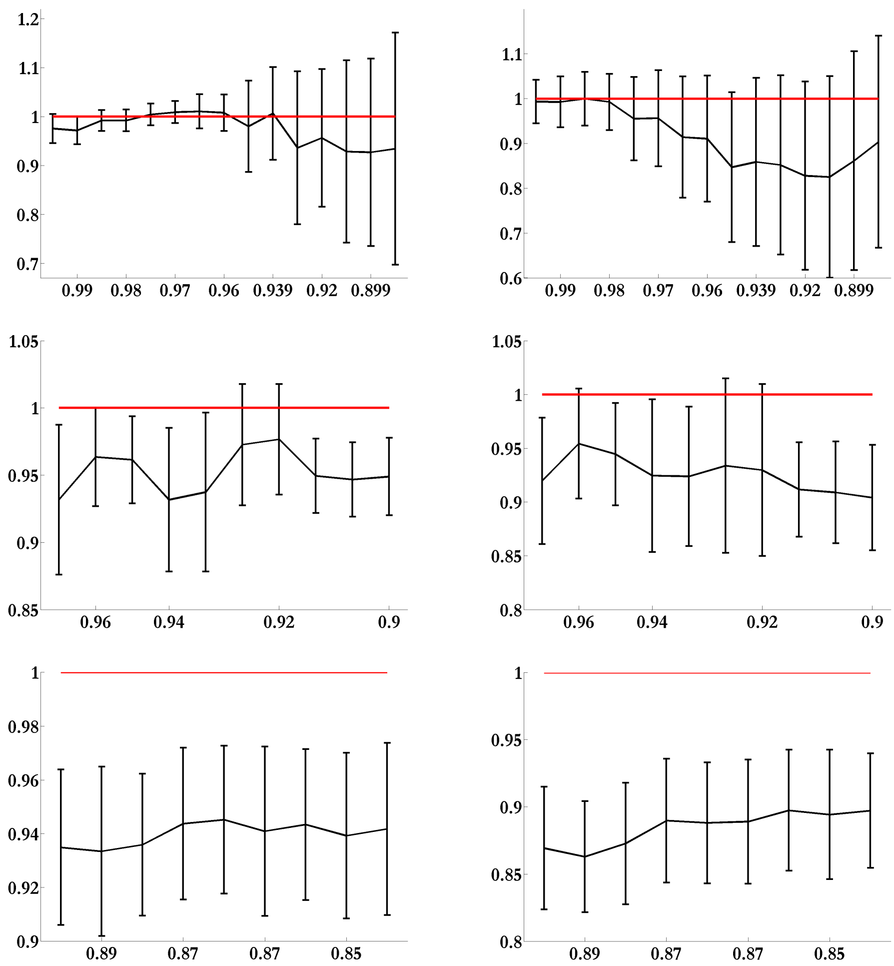

Fig. 2. Left column, horizon forecasting 5-day. Right column horizon forecasting 20-day. First row: region 1, i.e. $\alpha_1 + \beta_1 \approx 0.999$. Second row: region 2, i.e. $\alpha_1 + \beta_1 \approx 0.97$. Third row: region 3, i.e $\alpha_1 + \beta_1 \approx 0.9$. x-axis is always β_1, while y-axis is always the ratio MSE^{GARCH}/MSE^{MHF}

observation does not hold for real data series.

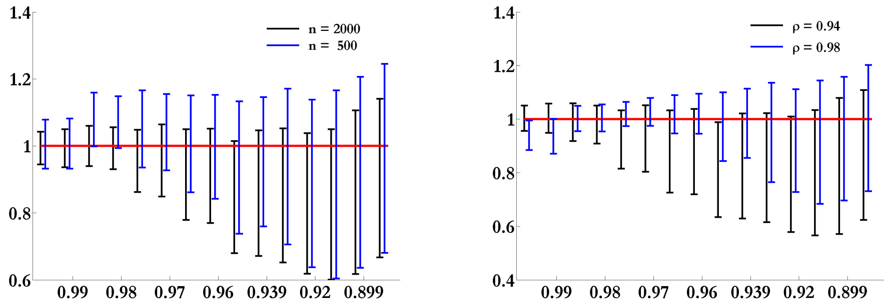

Fig. 3. In the left plot, GARCH(1,1) model is estimated on sample of two different lengths: 500 points in blue and 2000 points in black. In the right panel the *Mixed Historical Formula* is estimated, for two different values of ρ: 0.94 in blue and 0.98 in black.

The ratio MSE^{GARCH}/MSE^{MHF} depends on the number of points n used to estimate the GARCH model as well as on the value of ρ. The former, when n decreases, causes an increase and an upper shift in the empirical confidence bands (see left panel in figure 3). The latter introduces a very different type of behavior. As can be seen in figure 3 (right panel), when $\rho = 0.98$ the GARCH(1,1) model's performance deteriorates its performance as β_1 decreases, and vice versa for $\rho = 0.94$. In both case, for given values of β_1, the MHF model produces better results than the GARCH(1,1) model. It seems that a value of ρ that is the best in all cases does not exist. In fact, the results depend heavily on the parameters chosen for the simulation. We shall elaborate more on the subject when we present results on real data.

The results do not depend on the loss function used. If we use the ratio of Mean Absolute Error, instead of the Mean Squared Error, the range of percentiles reduces, but the conclusions remain the same.

5.2 Random Walk simulations

We perform simulations in which the price follows a random walk with null mean. The set-up is similar to the ARCH models, but the number of paths is greater (5000), since it is computationally less time-expensive.
The results are presented in figure 4. The ratio is always lower than unity, i.e. if the DGP is a Random Walk, the Moving Average is the best predictor. Also, the longer the sample, the better the forecast. Actually, when $n = 2000$, the confidence bands are always lower than the forecast obtained from $n = 500$. Another interesting feature is that, when ρ increases, (i.e the short volatility assumes a bigger weight in new formula), the Moving Average always produces better results. This is even more apparent in the case of longer maturity τ (left panel in figure 4).

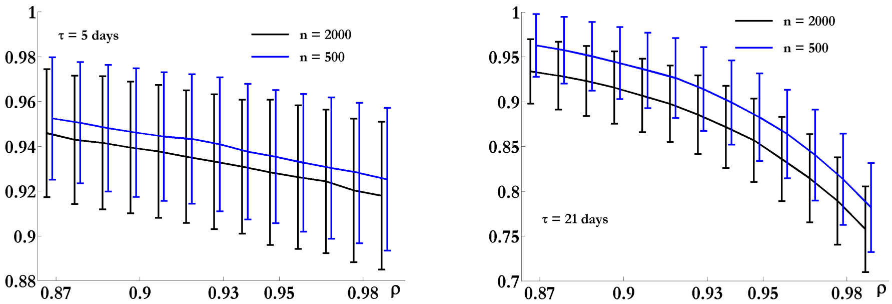

Fig. 4. In the left panel, ratio MSE^{MA}/MSE^{MHF} on horizon forecasting of 5 days. In the right panel the horizon forecasting is 20 days. Black line, Moving Average is estimated from a sample of 2000 days. Blue line, Moving Average is estimated from a sample of 500 days.

6 Real data

We apply the new formula to 75 daily data series: 36 Stock returns (from the American Market), 10 market indexes (from all the World), 15 American fixed income rates, 14 American foreign exchange rates. The data is listed in table 9.

The experimental set-up is analogous to that of the simulated analysis. The paths are 5000 points long (about twenty years). The benchmark models (ARCH and MA) are estimated using a rolling window of $n = 2000$ points with step 5 days. We assessed the performance of the proposed formula with ρ belonging to the subset $[0.87, 0.99]$. We shall now show the results relative to the Stock series in great detail, while we shall draw the main conclusion for the remaining three sets.

6.1 Assets data

The proposed formula does not work with asset data, for any choice of ρ. The results are reported in table 1 for a horizon of 5 days and in table 2 for a horizon of 20 days. The table entries show the percentage of how many times the ratio MSE^{Model}/MSE^{MHF} is greater than unity, i.e how many times the proposed formula produces better results. It is clear that such percentages are always inferior to 50%.

The seven models chosen are remarkable: in the ARCH model we know that the larger the set of data in the sample, the better the convergence. But the larger the data sample, the easier it is to find an I-GARCH effect. We tried to estimate the performance of the model using two different sample lengths: 1000 points and 2000. Assuming the series is stationary, we can observe that when the sample is longer, the estimation based on the Moving Average is

better. However, because the proposed formula relies on a Moving Average of 500 points, we shall consider this case as benchmark.
From the tables, we conclude that the best model for long horizons should be the Moving Average evaluated using the last 500 points, while nothing conclusive can be said for short horizons.

ρ \ Model	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.87	36%	39%	17%	19%	100%	31%	47%
0.88	36%	39%	17%	19%	100%	31%	47%
0.89	33%	33%	17%	19%	100%	28%	44%
0.9	33%	33%	17%	19%	100%	28%	44%
0.91	33%	33%	17%	19%	100%	28%	44%
0.92	33%	31%	17%	19%	100%	28%	44%
0.93	33%	31%	17%	19%	94%	28%	42%
0.94	33%	31%	17%	19%	17%	28%	42%
0.95	31%	31%	17%	19%	14%	28%	42%
0.96	31%	31%	17%	19%	8%	28%	42%
0.97	31%	31%	17%	19%	8%	28%	42%
0.98	31%	31%	17%	19%	6%	28%	42%
0.99	31%	31%	14%	17%	0%	28%	42%

Table 1
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for Stocks on 5-day horizon.

6.2 Fixed Income data

Results for Fixed Income data are reported in tables 3 and 4. The proposed formula gives the best results with respect to all other models, if $\rho \approx 0.88$. Such value of ρ results in weighting the short variance 80% on a 5-day horizon (see equation 4) and 35% on a 20-day horizon. We observe that there is little difference if the value $\rho = 0.89$ as opposed to $\rho = 0.87$ is used. Any value in this range results in a good forecast inside the error bands.
During our simulations, we observed that if we use the ARCH models as Data Generating Process models, the ratio of the 95^{th} percentile is seldom greater than 1.5. Surprisingly, we find that the percentage of ratio greater than this value is 60% for a 5-day horizon and 67% for a 20-day horizon FI. This means

ρ \ Model	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.87	50%	61%	22%	22%	100%	47%	61%
0.88	50%	61%	22%	19%	100%	47%	61%
0.89	50%	58%	22%	19%	100%	47%	61%
0.9	50%	56%	17%	19%	100%	44%	61%
0.91	50%	56%	17%	19%	100%	44%	58%
0.92	44%	53%	17%	19%	100%	44%	58%
0.93	44%	50%	17%	19%	100%	44%	56%
0.94	42%	50%	17%	19%	100%	42%	53%
0.95	36%	44%	17%	19%	100%	39%	50%
0.96	31%	39%	17%	19%	100%	39%	50%
0.97	31%	36%	11%	19%	100%	39%	44%
0.98	31%	33%	11%	19%	100%	36%	44%
0.99	31%	33%	8%	17%	8%	33%	44%

Table 2
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for Stocks on 20-day horizon.

that, with high probability, the GARCH(1,1) model and GJR-GARCH(1,1,1) model are not the true DGP. This conclusion is highly probable, yet not certain, since not all of the space parameters were mapped to obtain the true p-Value of a hypothesis test.

ρ \ Model	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.87	100%	100%	93%	93%	100%	100%	100%
0.88	100%	100%	93%	93%	100%	100%	100%
0.89	100%	100%	87%	93%	100%	100%	100%
0.9	100%	100%	87%	93%	100%	100%	100%

Table 3
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for Fixed Income data on 5-day horizon.

ρ \ Model	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.87	100%	100%	93%	93%	100%	100%	100%
0.88	100%	100%	93%	93%	100%	100%	100%
0.89	100%	100%	87%	93%	100%	100%	100%
0.9	100%	100%	87%	93%	100%	100%	100%

Table 4
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for Fixed Income data on 20-day horizon.

6.3 Foreign Exchange data

The results are reported in tables 5 and 6. We found two different values of ρ for the short and long horizons. For the short horizon, the value we recommend is $\rho \approx 0.9$, i.e. the short variance is weighted at about 80%. For the longer horizon, the choice of ρ is ≈ 0.96, i.e. the short variance weights for about 70%. As a consequence, the last 70 days carry more information than the last two years.
As for Fixed Income data, in Foreign Exchange data, we observe that the percentage of the ratio being greater than 1.5 is 71% for a 5-day forecast, and 93% for a 20-day forecast.

ρ \ Model	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.89	100%	100%	93%	100%	93%	100%	100%
0.9	100%	100%	93%	100%	93%	100%	100%
0.91	100%	100%	93%	100%	93%	100%	100%

Table 5
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for Foreign Exchange on 5-day horizon.

6.4 Market index data

Results on market index data are reported in tables 7 and 8. Here the ρ value for the short and the long horizons is always around 0.92. In the case of the short horizon, this means that the short variance is weighted at 85%, while for the long horizon only 50%. We observe that, for a 20-day forecast horizon, the ratio is always bigger than 1.5. This proves that the proposed formula provides a good quality forecast for market index series.

Model / ρ	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.94	93%	100%	93%	100%	93%	100%	100%
0.95	93%	100%	93%	100%	93%	100%	100%
0.96	93%	100%	93%	100%	93%	100%	100%
0.97	93%	100%	93%	100%	93%	100%	100%
0.98	93%	100%	93%	100%	93%	100%	100%

Table 6
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for Foreign Exchange on 20-day horizon.

Model / ρ	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.92	100%	100%	80%	90%	100%	100%	100%
0.93	100%	100%	80%	90%	100%	100%	100%

Table 7
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for market index data on 5-day horizon.

Model / ρ	garch 1000	garch 2000	ma 500	ma 2000	ewma	gjr-garch 1000	gjr-garch 2000
0.92	100%	100%	90%	90%	90%	100%	100%

Table 8
Percentage of times that MSE^{Model}/MSE^{MHF} is greater than unity for market index data on 20-day horizon.

7 Conclusions

In this paper we have presented a new formula to forecast conditional variance on a short horizon up to 20 days). Its application to real financial data shows that it produces excellent results in predicting variance for foreign exchange rates, interest rates and market indexes, while it generates poor results for stock returns. The models used for benchmarking are the ARCH models and the Moving Average model. These models have been selected due to them being both well-known and widely-accepted.
Interestingly, the results do not change if we use a different loss function (Mean Absolute Error), instead of the Mean Squared Error (MSE). This observation is relevant, because the MAE is less affected by outliers and we speculate that our proposed formula produces better results because the DGP is not one of the already-known parametrics.

If an economic loss function were to be used and different estimates of volatility to an hedging strategy were applied, the final pay-offs would not have shown noticeable differences. The reason lies in the fact that volatility plays a crucial role in pricing, but pricing functions are concave with respect to volatility. So the differences in volatility are reduced in price. This point will be the subject of further research by the author.

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8 Appendixes

Series	Type	From	To	Source	Codex
Alcoa Inc.	As	11-Oct-1985	04-Aug-2005	Yahoo	AA
Amer. Electr. Power	As	11-Oct-1985	05-Aug-2005	Yahoo	AEP
Amr Corp.	As	04-Oct-1985	05-Aug-2005	Yahoo	AMR
Boeing	As	14-Oct-1985	05-Aug-2005	Yahoo	BA
Burlington N.S.Fe Corp.	As	11-Oct-1985	04-Aug-2005	Yahoo	BNI
Citygroup	As	14-Oct-1985	05-Aug-2005	Yahoo	C
Caterpillar	As	14-Oct-1985	05-Aug-2005	Yahoo	CAT
CNF	As	04-Oct-1985	05-Aug-2005	Yahoo	CNF
Centerpoint	As	04-Oct-1985	05-Aug-2005	Yahoo	CNP
CSX	As	14-Oct-1985	05-Aug-2005	Yahoo	CSX
Delta Transportation	As	04-Oct-1985	05-Aug-2005	Yahoo	DAL
Du Pont De Nemours	As	14-Oct-1985	05-Aug-2005	Yahoo	DD
Walt Disney	As	14-Oct-1985	05-Aug-2005	Yahoo	DIS
Edison International	As	14-Oct-1985	05-Aug-2005	Yahoo	EIX
Federal Express	As	04-Oct-1985	05-Aug-2005	Yahoo	FDX

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Series	Type	From	To	Source	Codex
General Electrics	As	11-Oct-1985	04-Aug-2005	Yahoo	GE
General Motors	As	14-Oct-1985	05-Aug-2005	Yahoo	GM
Honeywell	As	14-Oct-1985	05-Aug-2005	Yahoo	HON
Hewlett Packard	As	14-Oct-1985	05-Aug-2005	Yahoo	HPQ
Intel Business Mach	As	14-Oct-1985	05-Aug-2005	Yahoo	IBM
Johnson Johnson	As	14-Oct-1985	05-Aug-2005	Yahoo	JNJ
JP Morgan	As	11-Oct-1985	05-Aug-2005	Yahoo	JPM
Coca Cola	As	14-Oct-1985	05-Aug-2005	Yahoo	KO
Southwest Airlines	As	14-Oct-1985	05-Aug-2005	Yahoo	LUV
McDonald	As	04-Oct-1985	05-Aug-2005	Yahoo	MCD
3M Corp	As	14-Oct-1985	05-Aug-2005	Yahoo	MMM
Merck	As	14-Oct-1985	05-Aug-2005	Yahoo	MRK
Norfolk Southern	As	04-Oct-1985	05-Aug-2005	Yahoo	NSC
Procter Gamble	As	14-Oct-1985	05-Aug-2005	Yahoo	PG
Ryder System	As	04-Oct-1985	05-Aug-2005	Yahoo	R
Union Pacific	As	14-Oct-1985	05-Aug-2005	Yahoo	UNP
United Tech. Corp.	As	14-Oct-1985	05-Aug-2005	Yahoo	UTX
Verizon Communic.	As	14-Oct-1985	05-Aug-2005	Yahoo	VZ
Wal-Mart Stores	As	14-Oct-1985	05-Aug-2005	Yahoo	WMT
Exxon-Mobil	As	14-Oct-1985	05-Aug-2005	Yahoo	XOM
Euro-Dollar - 3 Months	FI	03-Jan-1986	22-Jul-2005	Fed. Res.	ED3M
Euro-Dollar - 6 Months	FI	03-Jan-1986	22-Jul-2005	Fed. Res.	ED6M
5-Year Treasury note	FI	12-Aug-1985	05-Aug-2005	Yahoo	F̂VX
13-week Treasury bill	FI	12-Aug-1985	05-Aug-2005	Yahoo	ÎRX
Tr. bill secon. mkt -3M	FI	25-Jul-1985	22-Jul-2005	Fed. Res.	TBSM3M
Tr. bill secon. mkt -6M	FI	25-Jul-1985	22-Jul-2005	Fed. Res.	TBSM6M

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Series	Type	From	To	Source	Codex
Tr. Constant Mat. -10Y	FI	25-Jul-1985	22-Jul-2005	Fed. Res.	TCM10Y
Tr. Constant Mat. -1Y	FI	25-Jul-1985	22-Jul-2005	Fed. Res.	TCM1Y
Tr. Constant Mat. -3Y	FI	25-Jul-1985	22-Jul-2005	Fed. Res.	TCM3Y
Tr. Constant Mat. -7Y	FI	25-Jul-1985	22-Jul-2005	Fed. Res.	TCM7Y
Tr. note -10Y	FI	15-Aug-1985	05-Aug-2005	Yahoo	$\hat{T}$NX
Tr. bond -30Y	FI	13-Aug-1985	05-Aug-2005	Yahoo	$\hat{T}$YX
Cert. of Deposit -3M	FI	30-Oct-1975	22-Jul-2005	Fed. Res.	CD3M
Cert. of Deposit -6M	FI	10-Feb-1969	22-Jul-2005	Fed. Res.	CD6M
Federal Funds	FI	11-Sep-1985	22-Jul-2005	Fed. Res.	FEDFUND
USD/Australia	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
USD/Canada	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
USD/Denmark	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
USD/France	FE	02-Feb-1979	31-Dec-1998	Fed. Res.	
USD/Germany	FE	02-Feb-1979	31-Dec-1998	Fed. Res.	
USD/Ireland	FE	02-Feb-1979	31-Dec-1998	Fed. Res.	
USD/Italy	FE	01-Feb-1979	31-Dec-1998	Fed. Res.	
USD/Japan	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
USD/Norway	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
USD/Austria	FE	02-Feb-1979	31-Dec-1998	Fed. Res.	
USD/Spain	FE	01-Feb-1979	31-Dec-1998	Fed. Res.	
USD/Sweden	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
USD/Switzerland	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
USD/United Kingdom	FE	18-Sep-1985	05-Aug-2005	Fed. Res.	
All Ordinaries	Ind	05-Nov-1985	05-Aug-2005	Yahoo	AORD
Dow Jones Comp. Ind	Ind	11-Oct-1985	05-Aug-2005	Yahoo	DJA
Dow Jones Indus. Av.	Ind	14-Oct-1985	05-Aug-2005	Yahoo	DJI

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Series	Type	From	To	Source	Codex
Dow Jones Transp. Av.	Ind	11-Oct-1985	05-Aug-2005	Yahoo	DJT
Dow Jones Util. Av.	Ind	11-Oct-1985	05-Aug-2005	Yahoo	DJU
FTSE 100	Ind	23-Oct-1985	05-Aug-2005	Yahoo	FTSE
Standard&Poor 500	Ind	14-Oct-1985	05-Aug-2005	Yahoo	GSPC
Nasdaq	Ind	14-Oct-1985	05-Aug-2005	Yahoo	IXIC
Nikkei	Ind	09-Apr-1985	05-Aug-2005	Yahoo	N225
Standard&Poor 100	Ind	14-Oct-1985	05-Aug-2005	Yahoo	OEX
Major Market Index	Ind	08-Oct-1985	05-Aug-2005	Yahoo	XMI

Table 9. This table spans several pages. FI: Fixed Income. FE: Foreign Exchange. Ind: Index. As: Asset. USD: United States Dollar.